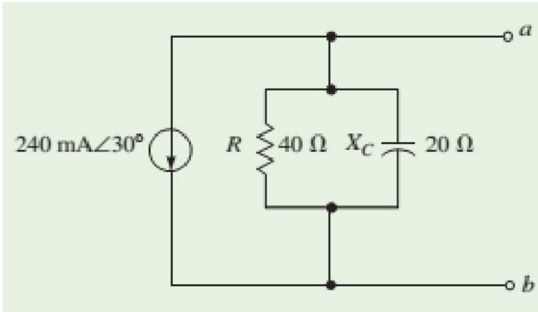
Benha University
Faculty of Engineering
Shoubra

Electrical Circuits (2)

Electrical Eng. Dept.
1st year communication
2014-2015

Sheet (1) solution... Review

1. Convert the current source of Figure (1) into an equivalent voltage source.



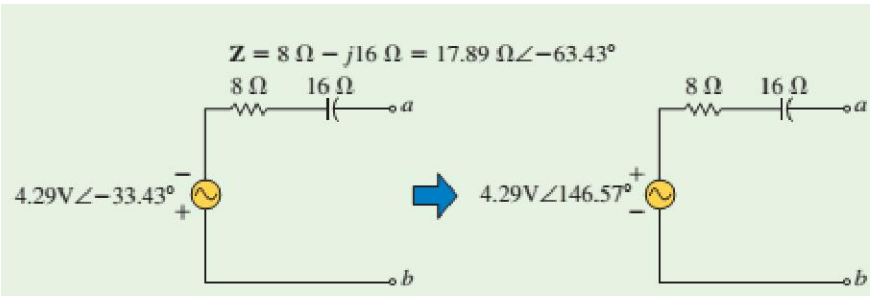
$$\mathbf{Z} = \frac{(40\ \Omega\angle 0^\circ)(20\ \Omega\ \angle -90^\circ)}{40\ \Omega - j20\ \Omega}$$

$$= \frac{800\ \Omega\ \angle -90^\circ}{44.72\angle -26.57^\circ}$$

$$= 17.89\ \Omega\angle -63.43^\circ = 8\ \Omega - j16\ \Omega$$

$$\mathbf{E} = (240\ \text{mA}\angle 30^\circ)(17.89\ \Omega\angle -63.43^\circ)$$

$$= 4.29\ \text{V}\angle -33.43^\circ$$



2. Given the circuit of Figure (2), write the loop equations and solve for the loop currents.

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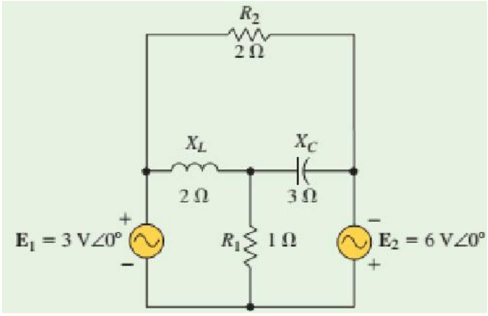
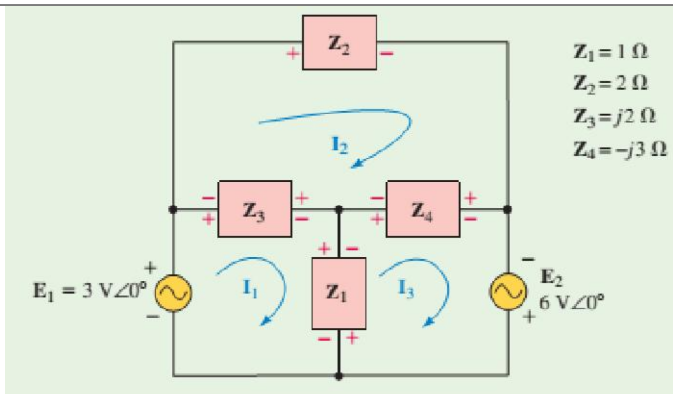


Figure (2)



The loop equations may now be written as

Loop 1: $(\mathbf{Z}_1 + \mathbf{Z}_3)\mathbf{I}_1 - (\mathbf{Z}_3)\mathbf{I}_2 - (\mathbf{Z}_1)\mathbf{I}_3 = \mathbf{E}_1$

Loop 2: $-(\mathbf{Z}_3)\mathbf{I}_1 + (\mathbf{Z}_2 + \mathbf{Z}_3 + \mathbf{Z}_4)\mathbf{I}_2 - (\mathbf{Z}_4)\mathbf{I}_3 = 0$

Loop 3: $-(\mathbf{Z}_1)\mathbf{I}_1 - (\mathbf{Z}_4)\mathbf{I}_2 + (\mathbf{Z}_1 + \mathbf{Z}_4)\mathbf{I}_3 = \mathbf{E}_2$

Using the given impedance values, we have

Loop 1: $(1\ \Omega + j2\ \Omega)\mathbf{I}_1 - (j2\ \Omega)\mathbf{I}_2 - (1\ \Omega)\mathbf{I}_3 = 3\ \text{V}$

Loop 2: $-(j2\ \Omega)\mathbf{I}_1 + (2\ \Omega - j1\ \Omega)\mathbf{I}_2 - (-j3\ \Omega)\mathbf{I}_3 = 0$

Loop 3: $-(1\ \Omega)\mathbf{I}_1 - (-j3\ \Omega)\mathbf{I}_2 + (1\ \Omega - j3\ \Omega)\mathbf{I}_3 = 6\ \text{V}$

Notice that in the preceding equations, the phase angles ($\theta = 0°$) for the voltages have been omitted. This is because $3\ \text{V}\angle 0° = 3\ \text{V} + j0\ \text{V} = 3\ \text{V}$.

Using a calculator such as the Sharp EL-506W, we have:

$$\mathbf{I}_1 = 5.26\text{A}\ \angle -41.25°$$
$$\mathbf{I}_2 = 3.18\text{A}\ \angle -45°$$
$$\mathbf{I}_3 = 3.39\text{A}\ \angle -1.03°$$

3. Use nodal analysis to determine the voltage V for the circuit of Figure (3).

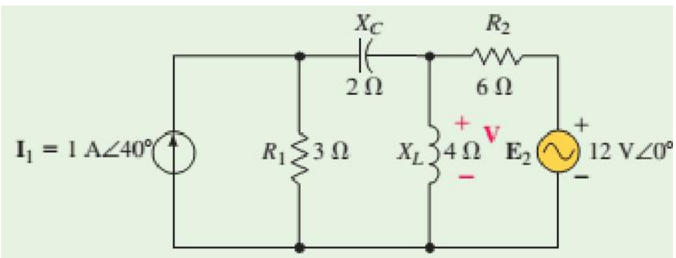


Figure (3)

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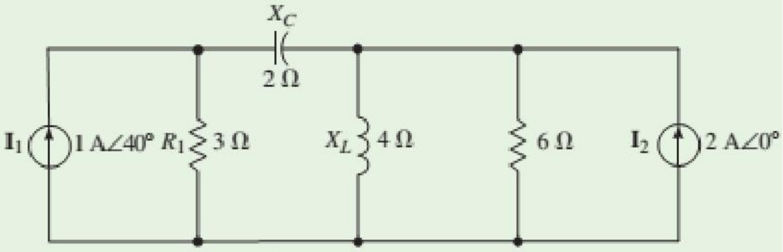
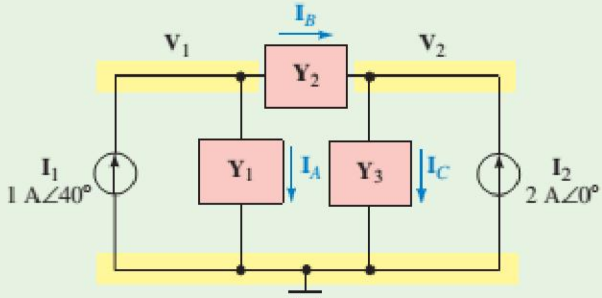


$$I_2 = \frac{E_2}{6\,\Omega} = \frac{12\text{ V}\angle 0^\circ}{6\,\Omega} = 2\text{ A}\angle 0^\circ$$



$$Y_1 = \tfrac{1}{3}\text{ S} + j\,0$$
$$Y_2 = 0 + j\tfrac{1}{2}\text{ S}$$
$$Y_3 = \tfrac{1}{6}\text{ S} - j\tfrac{1}{4}\text{ S}$$

Node 1:
$$I_A + I_B = I_1$$
$$Y_1V_1 + Y_2(V_1 - V_2) = I_1$$
$$(Y_1 + Y_2)V_1 - Y_2V_2 = I_1$$

Node 2:
$$I_C = I_B + I_2$$
$$Y_3V_2 = Y_2(V_1 - V_2) + I_2$$
$$-Y_2V_1 + (Y_2 + Y_3)V_2 = I_2$$

Now, substituting the values of admittance into the nodal equations, we get

Node 1: $$\left(\frac{1}{3} + \frac{1}{-j2}\right)V_1 - \left(\frac{1}{-j2}\right)V_2 = 1\angle 40^\circ$$

Node 2: $$-\left(\frac{1}{-j2}\right)V_1 + \left(\frac{1}{-j2} + \frac{1}{j4} + \frac{1}{6}\right)V_2 = 2\angle 0^\circ$$

Node 1: $$\left(\frac{1}{3} + j\frac{1}{2}\right)V_1 - \left(j\frac{1}{2}\right)V_2 = 1\angle 40^\circ$$

Node 2: $$-\left(j\frac{1}{2}\right)V_1 + \left(\frac{1}{6} + j\frac{1}{4}\right)V_2 = 2\angle 0^\circ$$

$$V_1 = 5.29\text{ V}\angle 48.75^\circ$$

$$V_2 = 5.80\text{ V}\angle 33.27^\circ$$

4. Determine the Y equivalent of the Δ network shown in Figure (4), show how to redistribute the Δ network to return in the form of Y.

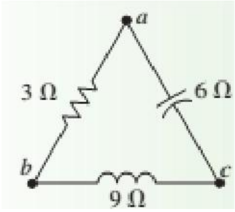
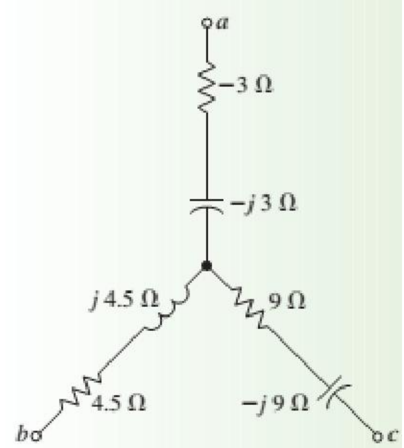


Figure (4)



$$\mathbf{Z}_1 = \frac{(3\ \Omega)(-j6\ \Omega)}{3\ \Omega - j6\ \Omega + j9\ \Omega} = \frac{-j18\ \Omega}{3 + j3} = \frac{18\ \Omega\angle -90^\circ}{4.242\angle 45^\circ}$$
$$= 4.242\ \Omega\angle -135^\circ$$
$$= -3.0\ \Omega - j3.0\ \Omega$$

$$\mathbf{Z}_2 = \frac{(3\ \Omega)(j9\ \Omega)}{3\ \Omega - j6\ \Omega + j9\ \Omega} = \frac{j27\ \Omega}{3 + j3} = \frac{27\ \Omega\angle 90^\circ}{4.242\angle 45^\circ}$$
$$= 6.364\ \Omega\angle 45^\circ$$
$$= 4.5\ \Omega + j4.5\ \Omega$$

$$\mathbf{Z}_3 = \frac{(j9\ \Omega)(-j6\ \Omega)}{3\ \Omega - j6\ \Omega + j9\ \Omega} = \frac{54\ \Omega}{3 + j3} = \frac{54\ \Omega\angle 0^\circ}{4.242\angle 45^\circ}$$
$$= 12.73\ \Omega\angle -45^\circ$$
$$= 9.0\ \Omega - j9.0\ \Omega$$

The second requirement … self study

5. Consider the circuit of Figure (5), Find VR and VC using the superposition theorem.

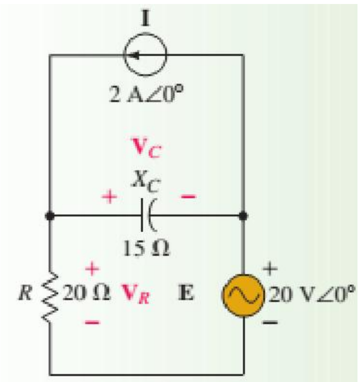


Figure (5)

The superposition theorem may be employed as follows:

Voltages due to the current source: Eliminating the voltage source, we obtain the circuit shown in Figure

The impedance "seen" by the current source will be the parallel combination of $\mathbf{R}\|\mathbf{Z}_C$.

$$\mathbf{Z}_1 = \frac{(20\ \Omega)(-j15\ \Omega)}{20\ \Omega - j15\ \Omega} = \frac{300\ \Omega\angle-90^\circ}{25\ \Omega\angle-36.87^\circ} = 12\ \Omega\angle-53.13^\circ$$

The voltage $\mathbf{V}_{R(1)}$ is the same as the voltage across the capacitor, $\mathbf{V}_{C(1)}$. Hence,

$$\begin{aligned}\mathbf{V}_{R(1)} &= (2\ \text{A}\angle0^\circ)(12\ \Omega\angle-53.13^\circ)\\ &= 24\ \text{V}\angle-53.13^\circ\end{aligned}$$

Voltages due to the voltage source: Eliminating the current source, we have the circuit shown in Figure

The voltages $\mathbf{V}_{R(2)}$ and $\mathbf{V}_{C(2)}$ are determined by applying the voltage divider rule,

$$\begin{aligned}\mathbf{V}_{R(2)} &= \frac{20\ \Omega\angle0^\circ}{20\ \Omega - j15\ \Omega}(20\ \text{V}\angle0^\circ)\\ &= \frac{400\ \text{V}\angle0^\circ}{25\ \angle-36.87^\circ} = 16\ \text{V}\angle+36.87^\circ\end{aligned}$$

$$\begin{aligned}\mathbf{V}_R &= \mathbf{V}_{R(1)} + \mathbf{V}_{R(2)}\\ &= 24\ \text{V}\angle-53.13^\circ + 16\ \text{V}\angle36.87^\circ\\ &= (14.4\ \text{V} - j19.2\ \text{V}) + (12.8\ \text{V} + j9.6\ \text{V})\\ &= 27.2\ \text{V} - j9.6\ \text{V}\\ &= 28.84\ \text{V}\angle-19.44^\circ\end{aligned}$$

6. Determine the Thévenin equivalent circuit external to ZL in the circuit in Figure (6)

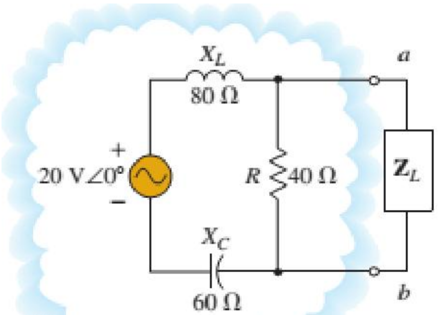
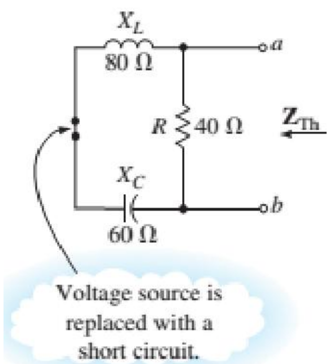
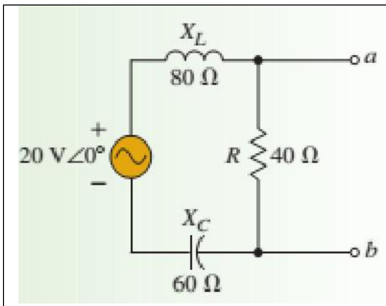


Figure (6)



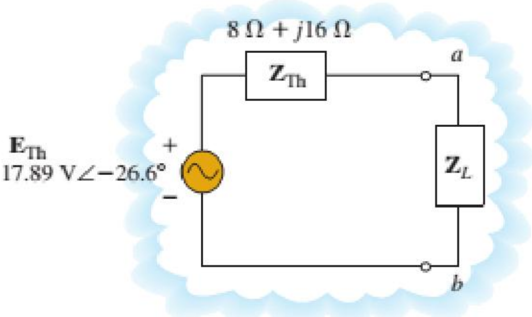
$$\begin{aligned}\mathbf{Z}_{Th} &= \mathbf{R}\|(\mathbf{Z}_L + \mathbf{Z}_C)\\ &= \frac{(40\ \Omega\angle0^\circ)(20\ \Omega\angle90^\circ)}{40\ \Omega + j20\ \Omega}\\ &= \frac{800\ \Omega\angle90^\circ}{44.72\ \Omega\angle26.57^\circ}\\ &= 17.89\ \Omega\angle63.43^\circ\\ &= 8\ \Omega + j16\ \Omega\end{aligned}$$



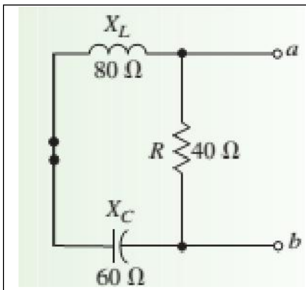
$$\mathbf{E}_{Th} = \mathbf{V}_{ab} = \frac{40\ \Omega\angle 0^\circ}{40\ \Omega + j80\ \Omega - j60\ \Omega}(20\ \text{V}\angle 0^\circ)$$

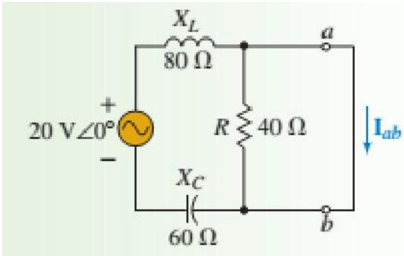
$$= \frac{800\ \text{V}\angle 0^\circ}{44.72\ \Omega\angle 26.57^\circ}$$

$$= 17.89\ \text{V}\angle -26.57^\circ$$



7. Repeat the previous problem but by using the Norton equivalent.





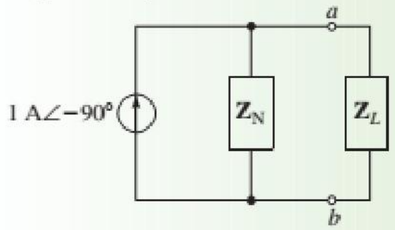
$$\mathbf{Z}_N = \frac{(40\ \Omega\angle 0^\circ)(20\ \Omega\angle 90^\circ)}{40\ \Omega + j20\ \Omega}$$

$$= \frac{800\ \Omega\angle 90^\circ}{44.72\angle 26.57^\circ}$$

$$= 17.89\ \Omega\angle 63.43^\circ$$

$$= 8\ \Omega + j16\ \Omega$$

$\mathbf{Z}_N = 8\ \Omega + j16\ \Omega = 17.89\ \Omega\angle 63.43^\circ$



$$\mathbf{I}_N = \mathbf{I}_{ab} = \frac{20\ \text{V}\angle 0^\circ}{j80\ \Omega - j60\ \Omega}$$

$$= \frac{20\ \text{V}\angle 0^\circ}{20\ \Omega\angle -90^\circ}$$

$$= 1.00\ \text{A}\angle -90^\circ$$

Good Luck

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